A New Algorithm for the Design of Linear Prediction Error Filters Using Cumulant-Based MSE Criteria

Chong-Yung Chi, Wen-Jie Chang, and Chih-Chun Feng

Abstract—**This correspondence proposes a new algorithm for the design of (minimum-phase) linear prediction error (LPE) filters using two new cumulant (higher order statistics) based MSE criteria when the given stationary random signal $x(n)$ is nonGaussian and contaminated by Gaussian noise. It is shown that the designed LPE filters based on the proposed criteria are identical to the conventional correlation (second-order statistics) based LPE filter as if $x(n)$ were noise-free measurements. As correlation-based LPE filters, coefficients of the designed cumulant-based LPE filters can be obtained by solving a set of symmetric Toeplitz linear equations using the well-known computationally efficient Levinson–Durbin recursion. Moreover, the proposed two criteria are applicable for any cumulant order $M \geq 3$, and one of the proposed criteria for $M = 3$ reduces to Delopoulos and Giannakis' third-order cumulant-based MSE criterion. Some simulation results are then provided to support the analytical results.**

I. Introduction

Linear prediction error (LPE) filters [1], [2] have been widely used in various signal processing areas such as speech processing, seismic deconvolution, and spectral estimation. The conventional LPE filter is based on the mean-square-error (MSE) criterion, and its coefficients can be obtained by solving a set of symmetric Toeplitz linear equations (the well-known Yule–Walker equations) formed of correlations $r_{xx}(k)$ of the stationary signal $x(n)$ of interest. However, correlation-based LPE filters are sensitive to measurement noise simply because $r_{xx}(k)$ includes correlations of noise.

Delopoulos and Giannakis [3] proposed a third-order cumulant-based MSE criterion and a fourth-order cumulant-based MSE criterion. It was shown in [3] that when the nonGaussian signal $x(n)$ is contaminated by Gaussian noise, their criteria are equivalent to the correlation-based MSE criterion as if $x(n)$ were a noise-free signal. Surely, these two criteria can be used to design LPE filters; moreover, the filter coefficients can be obtained by solving a set of linear equations when their third-order cumulant-based MSE criterion is used. Chi *et al.* [4]–[6] also proposed some cumulant-based criteria for the design of LPE filters when $x(n)$ is nonGaussian. Again, these cumulant-based criteria are also equivalent to the correlation-based MSE criterion as if $x(n)$ were a noise-free signal, but the filter coefficients can only be obtained using nonlinear optimization algorithms due to lack of closed-form solutions. This correspondence further proposes a new algorithm for the design of LPE filters using two new cumulant-based MSE criteria, and the filter coefficients can be obtained by solving a set of symmetric Toeplitz linear equations to which the Levinson–Durbin recursion can be applied. The proposed two criteria are applicable for any cumulant order $M \geq 3$. Moreover, one of the proposed two criteria for $M = 3$ reduces to Delopoulos and Giannakis' third-order cumulant-based MSE criterion mentioned above.

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The authors are with the Department of Electrical Engineering, National Tsing Hua University, Hsinchu, Taiwan, Republic of China.

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In Section II, some modeling assumptions followed by a brief review of conventional correlation-based LPE filters are presented. Section III presents the two new cumulant-based MSE criteria for the design of LPE filters together with some analytical results. Then, some simulation results are provided to support the proposed criteria in Section IV. Finally, we draw some conclusions.

II. Modeling Assumptions and Conventional LPE Filters

Assume that $x(n), n = 0, 1, \ldots, N-1$ are the given nonGaussian noisy measurements generated from the following convolutional model:

$$x(n) = u(n) * h(n) + w(n) \tag{1}$$

where $h(n)$ is a linear time-invariant (LTI) system, $u(n)$ is the driving input to the system, and $w(n)$ is measurement noise. Let us make the following assumptions for the model given by (1):

A1) The system $h(n)$ is causal and stable; it can be minimum phase or nonminimum phase.

A2) The input $u(n)$ is real, zero-mean, stationary, i.i.d., nonGaussian with variance σ_u^2 and Mth-order ($M \geq 3$) cumulant γ_M.

A3) Measurement noise $w(n)$ is Gaussian, which can be white or colored with unknown statistics.

A4) The input $u(n)$ is statistically independent of $w(n)$.

Let $v_p(n)$ be a pth-order causal FIR filter with $v_p(0) = 1$ driven by $x(n)$. Then, the output $e(n)$ (prediction error of $x(n)$) of the filter is given by

$$e(n) = x(n) * v_p(n) = x(n) + \sum_{i=1}^{p} v_p(i) x(n-i). \tag{2}$$

The conventional pth-order LPE filter is the $v_p(n)$ such that the mean square error $E[e^2(n)]$ is minimum. The resulting LPE filter is known to be a minimum-phase whitening filter, but it tries to whiten the noisy signal $x(n)$ instead of the noise-free signal $y(n) = u(n) * h(n)$.

III. New Cumulant-Based MSE Criteria for the Design of LPE Filters

The prediction error given by (2) can be further expressed as

$$e(n) = \xi(n) + w'(n) \tag{3}$$

where $w'(n) = w(n) * v_p(n)$ is also a Gaussian noise sequence since $w(n)$ is Gaussian, and

$$\xi(n) = u(n) * g(n) \tag{4}$$

is the noise-free output of the LPE filter where

$$g(n) = h(n) * v_p(n). \tag{5}$$

Moreover, it can be shown that the Mth-order cumulant function, which is denoted $C_{M,x}(k_1, k_2, \ldots, k_{M-1})$, of $x(n)$ is given by [7], [8]

$$C_{M,x}(k_1, k_2, \ldots, k_{M-1}) = \gamma_M \sum_{n=-\infty}^{\infty} h(n) h(n+k_1) \cdots h(n+k_{M-1}) \tag{6}$$

which also implies that

$$C_{M,\epsilon}(k_1,k_2,\ldots,k_{M-1}) = C_{M,\xi}(k_1,k_2,\ldots,k_{M-1}) = \gamma_M \sum_{n=-\infty}^{\infty} g(n)g(n+k_1)\cdots g(n+k_{M-1}). \quad (7)$$

The new cumulant-based MSE criteria for the design of LPE filters are described in the following theorem:

Theorem 1: Let $\widehat{v}_p(n)$ be the optimum $v_p(n)$ based on either of the following two criteria:

$$\widetilde{J}_M(v_p(n)) = \frac{1}{|V_p(z=1)|^{2(M-2)}} \times \left\{ \sum_{k_2=-\infty}^{\infty} \cdots \sum_{k_{M-1}=-\infty}^{\infty} C_{M,e}(k_1=0,k_2,\ldots,k_{M-1}) \right\}^2 \geq \widetilde{J}_M(\widehat{v}_p(n)) \quad (8)$$

$$J_M(v_p(n)) = \left\{ \sum_{k=-\infty}^{\infty} \mathrm{Cum}^{(M)}(e(n),e(n), x(n-k),\ldots,x(n-k)) \right\}^2 \geq J_M(\widehat{v}_p(n)) \quad (9)$$

where $M \geq 3$, and $V_p(z)$ is the $\mathcal{Z}$ transform of $v_p(n)$. Then, the $\widehat{v}_p(n)$ associated with $\widetilde{J}_M$ and the one associated with J_M are identical to the conventional pth-order LPE filter associated with the case of SNR $= \infty$, as long as $\gamma_M F_1 \neq 0$ for the former, and $\gamma_M F_{M-2} \neq 0$ for the latter, where

$$F_m = \sum_{n=-\infty}^{\infty} h^m(n). \quad (10)$$

Proof: It is sufficient to prove that minimizing $\widetilde{J}_M$ and J_M is equivalent to minimizing $E[\xi^2(n)]$, which is equal to $E[e^2(n)]$ when SNR $= \infty$ (see (3)). Let us simplify the numerator of $\widetilde{J}_M$ as follows:

$$\begin{aligned}
&\sum_{k_2=-\infty}^{\infty} \cdots \sum_{k_{M-1}=-\infty}^{\infty} C_{M,e}(0,k_2,\ldots,k_{M-1}) \\
&= \gamma_M \sum_{k_2=-\infty}^{\infty} \cdots \sum_{k_{M-1}=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} g^2(n)g(n+k_2)\cdots \times g(n+k_{M-1}) \quad \text{(since (7))} \\
&= \gamma_M \left\{ \sum_{n=-\infty}^{\infty} g^2(n) \right\} \left\{ \sum_{n=-\infty}^{\infty} g(n) \right\}^{M-2} \\
&= \frac{\gamma_M}{\sigma_u^2} E[\xi^2(n)][H(z=1)V_p(z=1)]^{M-2}. \quad (11)
\end{aligned}$$

Substituting (11) into $\widetilde{J}_M$ given by (8) yields

$$\widetilde{J}_M = \left[\frac{\gamma_M F_1^{M-2}}{\sigma_u^2}\right]^2 \{E[\xi^2(n)]\}^2. \quad (12)$$

On the other hand, J_M can be simplified as follows:

$$\begin{aligned}
J_M &= \left\{ \sum_{k=-\infty}^{\infty} \mathrm{Cum}^{(M)}(e(n),e(n), x(n-k),\ldots,x(n-k)) \right\}^2 \\
&= \left[\gamma_M \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} g^2(n)h^{M-2}(n-k) \right]^2 \quad \text{(since (A3), (A4), (6) and (7))} \\
&= \left[\frac{\gamma_M}{\sigma_u^2} \sum_{n=-\infty}^{\infty} h^{M-2}(n) \right]^2 \left[\sigma_u^2 \sum_{n=-\infty}^{\infty} g^2(n) \right]^2 \\
&= \left[\frac{\gamma_M F_{M-2}}{\sigma_u^2} \right]^2 \{E[\xi^2(n)]\}^2 = \rho_p^2(M) \quad (13)
\end{aligned}$$

where

$$\rho_p(M) = \left[\frac{\gamma_M F_{M-2}}{\sigma_u^2}\right] E[\xi^2(n)]. \quad (14)$$

One can see, from (12) and (13), that minimizing either $\widetilde{J}_M$ or J_M is equivalent to minimizing $E[\xi^2(n)]$ when $\gamma_M F_1 \neq 0$ for the former and $\gamma_M F_{M-2} \neq 0$ for the latter. **Q.E.D.**

Note that $\widetilde{J}_M = J_M = C_{2,e}^2(0) = \{E[e^2(n)]\}^2$ for $M = 2$, which indicates that conceptually, the proposed two criteria $\widetilde{J}_M$ and J_M can be viewed as a square of the sum of all higher order joint correlations of $e^2(n)$ and $\{e(n+k_2),\ldots,e(n+k_{M-1})\}$ and that of $e^2(n)$ and $(M-2)$ identical random variables $x(n-k)$, respectively. Note, from (9), that the proposed criterion J_M for $M = 3$ and $M = 4$ can be expressed as

$$J_3 = \left\{ \sum_{k=-\infty}^{\infty} E[e^2(n)x(n-k)] \right\}^2, \quad (15)$$

which is equivalent to the square of Delopoulos and Giannakis' [3] third-order cumulant-based MSE criterion, and

$$J_4 = \left\{ \sum_{k=-\infty}^{\infty} E[e^2(n)x^2(n-k)] - 2(E[e(n)x(n-k)])^2 - E[e^2(n)]E[x^2(n-k)] \right\}^2 \quad (16)$$

respectively. Delopoulos and Giannakis [3] also proposed a fourth-order cumulant-based MSE criterion, which is given as follows:

$$\begin{aligned}
J_{\mathrm{DG}}^{(4)} = \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} e^{j\omega_0(k_1-k_2)} &\{E[x(n+k_1)x(n+k_2)\cdot e^2(n)] \\
&-2E[x(n+k_1)e(n)]\cdot E[x(n+k_2)e(n)] \\
&-E[x(n+k_1)x(n+k_2)]\cdot E[e^2(n)]\}. \quad (17)
\end{aligned}$$

From (16) and (17), one can see that if the criterion $J_{\mathrm{DG}}^{(4)}$ given by (17) is modified by letting $k_1 = k_2$, then $J_4 = \{J_{\mathrm{DG}}^{(4)}\}^2$. In addition, note that J_3 and J_4 are always concave functions of $v_p(n)$, whereas $J_{\mathrm{DG}}^{(3)}$ and $J_{\mathrm{DG}}^{(4)}$ can be convex functions of $v_p(n)$, and that J_4 uses only a "1-dimensional slice" of fourth-order cross cumulants, whereas $J_{\mathrm{DG}}^{(4)}$ uses a "2-dimensional slice" of fourth-order cross cumulants. Moreover, the proposed criterion J_M always requires a 1-dimensional slice of Mth-order cumulants, and the designed LPE filter can be obtained by solving a set of linear symmetric Toeplitz equations given by (20) below for all $M \geq 3$. Therefore, the proposed criterion J_M is computationally much more practical than Delopoulos and Giannakis' criterion for $M \geq 4$. On the other hand, Theorem 1 also implies the following fact:

F1) Assume that $H(z)$ and $H_{\mathrm{MP}}(z)$ are spectrally equivalent, and $H_{\mathrm{MP}}(z)$ is minimum phase with $H_{\mathrm{MP}}(z=\infty) = 1$. Then, the optimum $\widehat{v}_p(n)$ (associated with either $\widetilde{J}_M$ or J_M) $\rightarrow \widehat{v}_p{}'(n)$ as $p \rightarrow \infty$ where $\widehat{v}_p{}'(n)$ is the impulse response of the inverse filter $1/H_{\mathrm{MP}}(z)$.

TABLE I
SIMULATION RESULTS OF EXAMPLE I

$a(1) = 0.7$, $a(2) = 0.1$, $N = 4096$, 30 independent runs					
Criterion	Estimated values (mean±standard deviation)				
		$SNR = \infty$	$SNR = 40$	$SNR = 10$	$SNR = 5$
MSE	$\widehat{v}(1)$	0.7023 ± 0.0168	0.6713 ± 0.0173	0.5975 ± 0.0178	0.5262 ± 0.0179
	$\widehat{v}(2)$	0.1009 ± 0.0164	0.0793 ± 0.0171	0.0312 ± 0.0178	-0.0093 ± 0.0181
$J_3(\widetilde{J}_3)$	$\widehat{v}(1)$	0.7031 ± 0.0507	0.7049 ± 0.0615	0.7046 ± 0.0886	0.7056 ± 0.1370
	$\widehat{v}(2)$	0.1003 ± 0.0392	0.0976 ± 0.0494	0.0937 ± 0.0714	0.0923 ± 0.1112
$J_4(\widetilde{J}_4)$	$\widehat{v}(1)$	0.6926 ± 0.0438	0.6874 ± 0.0452	0.6812 ± 0.0513	0.6760 ± 0.0602
	$\widehat{v}(2)$	0.0952 ± 0.0560	0.0904 ± 0.0551	0.0859 ± 0.0559	0.0857 ± 0.0594

Because the optimum LPE filters associated with $\widetilde{J}_M$ and J_M are identical, let us only present the way to obtain the optimum LPE filter based on the proposed criterion J_M given by (9). The criterion J_M can be further simplified by substituting (2) into (9) as follows:

$$J_M = \{\rho_p(M)\}^2 = \left\{\sum_{i=0}^{p}\sum_{j=0}^{p} v_p(i)v_p(j)c(j-i)\right\}^2 \text{ (since (13))} \tag{18}$$

where

$$c(i) = \sum_{k=-\infty}^{\infty} C_{M,x}(0,\ldots,0,k,k+i) = c(-i). \tag{19}$$

Setting the partial derivative of J_M given by (18) with respect to $v_p(i)$, $i = 1, 2, \ldots, p$ equal to zero, one can obtain a set of linear equations given as follows:

$$\mathbf{C}_{p+1}\widehat{\mathbf{v}}_p = \theta \tag{20}$$

where $\widehat{\mathbf{v}}_p = [1, \widehat{v}_p(1), \ldots, \widehat{v}_p(p)]^T$ and $\theta = [\rho_p(M), 0, \ldots, 0]^T$ are $(p+1) \times 1$ vectors, and $\mathbf{C}_{p+1}$ is a $(p+1) \times (p+1)$ symmetric Toeplitz matrix with the (i,j)th component given by

$$[\mathbf{C}_{p+1}]_{i,j} = c(i-j), \quad 1 \le i \le (p+1), \quad 1 \le j \le (p+1). \tag{21}$$

Therefore, $\widehat{\mathbf{v}}_p$ can be solved using the computationally efficient Levinson–Durbin recursion [1], [2]. Furthermore, taking the Fourier transform of $c(i)$ given by (19), one can easily show that

$$C(e^{j\omega}) = \sum_{i=-\infty}^{\infty} c(i)e^{-j\omega i} = \gamma_M F_{M-2}|H(e^{j\omega})|^2 \tag{22}$$

which implies the following fact:

F2) The sequence $c(i) = c(-i)$ is positive definite if $\gamma_M F_{M-2} > 0$ and negative definite if $\gamma_M F_{M-2} < 0$. Therefore, $c(i)$ can be thought of as a legitimate correlation sequence if $\gamma_M F_{M-2} > 0$ and that the desired $\widehat{v}_p(n)$ obtained by solving (20) is minimum phase [1], [2].

Moreover, $c(i)$ must be estimated from data in practice. For instance, $c(i)$ can be estimated as

$$\widehat{c}(i) = \sum_{k=-K}^{K} \widehat{C}_{M,x}(0,\ldots,0,k,k+i) \tag{23}$$

where $\widehat{C}_{M,x}(k_1, k_2, \ldots, k_{M-1})$ is the biased sample cumulant of $x(n)$, and the integer K must be chosen large enough such that $\widehat{c}(i)$ is approximate to $\sum_{k=-\infty}^{\infty} \widehat{C}_{M,x}(0,\ldots,0,k,k+i)$. Note that $\widehat{C}_{M,x}(0,\ldots,0,k,k+i)$ is known to be a consistent estimate [8] for $C_{M,x}(0,\ldots,0,k,k+i) = \gamma_M \sum_n h(n)^{M-2}h(n+k)h(n+k+i)$, which is nonzero for $-L < k < L - i$ (assuming $i \ge 0$), where L is the length of $h(n)$. Therefore, $\widehat{c}(i)$ is also a consistent estimate for $c(i)$ for $K \ge L - 1$.

IV. SIMULATION RESULTS

In this section, two simulation examples are to be presented to demonstrate that the proposed criteria can be used for the design of LPE filters. In these examples, $H(z)$ was a second-order AR model; the order of the LPE filter to be designed was $p = 2$, and filter coefficients were obtained by solving (20) in which $c(i)$ was replaced by $\widehat{c}(i)$ given by (23) with a proper value for parameter K; thirty independent runs for data length $N = 4096$ were performed to calculate statistical mean and standard deviation. For comparison, conventional LPE filters were also obtained using the well-known Burg's algorithm [1], [2].

Example 1: The driving input $u(n)$ used was a zero-mean, exponentially distributed i.i.d. random sequence with variance $\sigma_u^2 = 1$, $\gamma_3 = 2$, and $\gamma_4 = 6$. A second-order autoregressive (AR) model $H(z) = 1/A(z)$, where

$$A(z) = 1 + a(1)z^{-1} + a(2)z^{-2} = 1 + 0.7z^{-1} + 0.1z^{-2} \tag{24}$$

was used, and $w(n)$ was white Gaussian. The desired LPE filters associated with both $J_3(\widetilde{J}_3)$ and $J_4(\widetilde{J}_4)$ were obtained with $K = 5$ and $K = 10$ (see (23)), respectively.

The simulation results for SNR = 5, 10, 40, and ∞ are shown in Table I. Observe, from this table, that when SNR is large (SNR = ∞), mean values of all estimated filter coefficients are very close to the true AR parameters. When SNR is low (SNR=5), biases of estimated filter coefficients associated with the proposed cumulant-based MSE criteria are much smaller than those associated with the conventional MSE criterion, and mean square errors (sum of variance and square of bias) of estimated filter coefficients shown in the former are also smaller than those shown in the latter, although standard deviations of estimated filter coefficients for the latter are smaller than those for the former.

Example 2: The driving input used in this example was the same as that used in Example 1, and the system $h(n)$ was also a second-order AR model with $H(z) = 1/A(z)$, where

$$A(z) = 1 - z^{-1} + 0.34z^{-2}. \tag{25}$$

However, both the case of white Gaussian noise and the case of colored Gaussian noise were considered in the simulation. Colored Gaussian noise sequences were generated from a causal high-pass FIR filter with transfer function $B(z) = 1 - 1.2z^{-1} + 0.32z^{-2}$, where the input is a white Gaussian noise sequence. The criterion $J_3(\widetilde{J}_3)$ was used with parameter K set to 4 (see (23)) in solving the filter coefficients.

Instead of showing numerical results with a table as in Example 1, we present the simulation results for SNR = 5 via AR power spectral density (PSD) $1/|\widehat{V}_p(e^{j2\pi f})|^2$. The simulation results for the case of white Gaussian noise are shown in Fig. 1. Thirty PSD estimates associated with $J_3(\widetilde{J}_3)$ and those obtained using Burg's algorithm are shown in Fig. 1(a) and (b), respectively, in which different kinds of lines were used to make each single PSD estimate discernible from

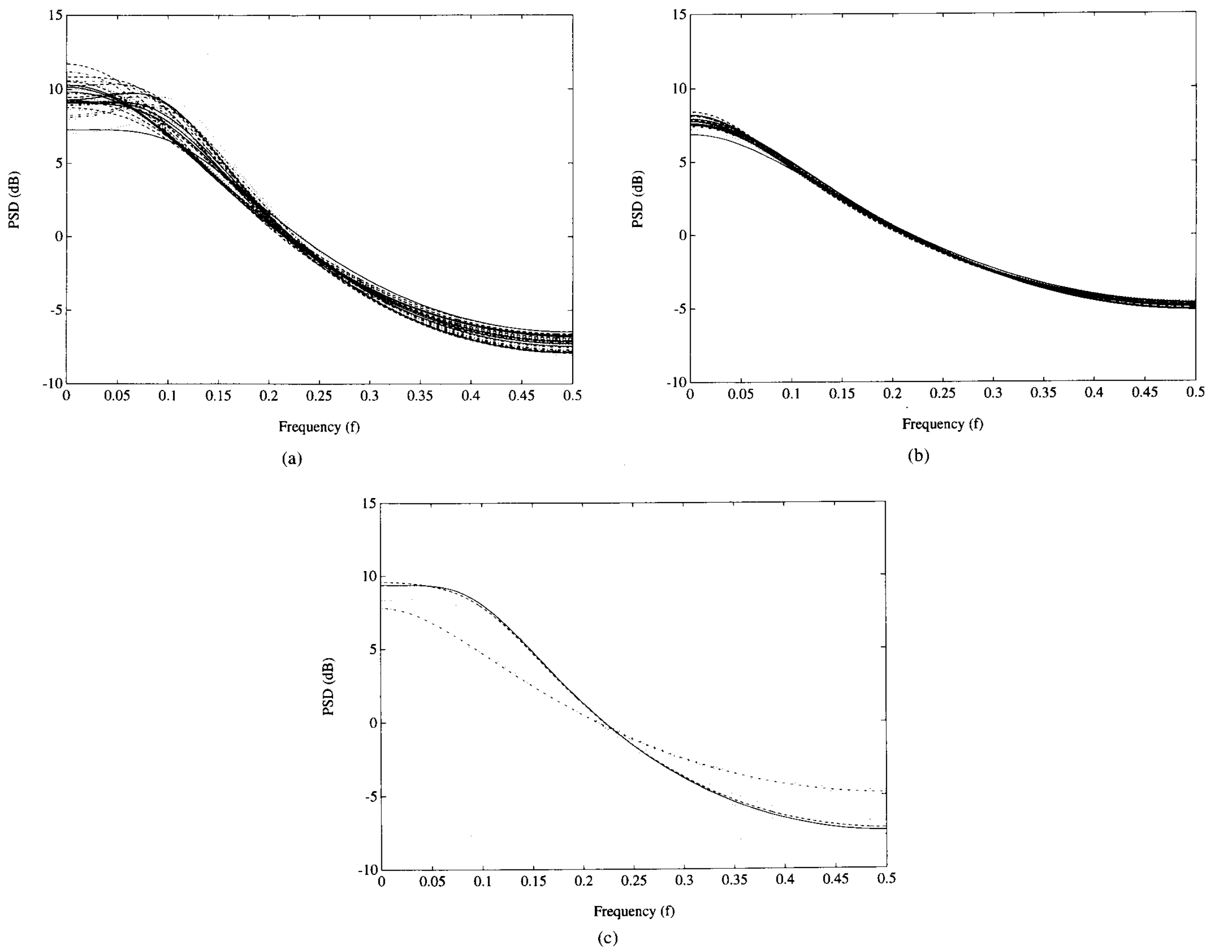

Fig. 1. Simulation results for the case of white Gaussian noise with SNR = 5: (a) Thirty PSD estimates obtained by the proposed criterion $J_3(\tilde{J}_3)$; (b) those obtained by Burg's algorithm; (c) averages depicted by dashed line and dashed-dotted line as well as $\pm$ standard deviation (dotted line) of the 30 estimates shown in (a) and (b), respectively, together with the true PSD (solid line).

other PSD estimates. One can see, from these two figures, that the variance of the AR spectral estimator associated with $J_3(\tilde{J}_3)$ is larger than that associated with Burg's algorithm. On the other hand, the respective averages of the 30 PSD estimates shown in Fig. 1(a) and (b) together with the true PSD (solid line) are shown in Fig. 1(c). Note, from this figure that the average (dashed line) of the 30 PSD estimates associated with $J_3(\tilde{J}_3)$ is quite close to the true PSD (within 0.2 dB), but that (dashed-dotted line) associated with Burg's algorithm departs from the true power spectrum by about 2 dB. In other words, the bias of the AR spectral estimator associated with $J_3(\tilde{J}_3)$ is much smaller than that associated with Burg's spectral estimator. The reason for this is simply because noise power spectrum is significant to correlation-based power spectral estimators for this case (SNR = 5).

The simulation results for the case of colored Gaussian noise corresponding to those shown in Fig. 1(a) through (c) are shown in Fig. 2(a) through (c), respectively. The same conclusion can be drawn from Fig. 2 as drawn from Fig. 1. Additionally, the average (dashed-dotted line) associated with Burg's algorithm shown in Fig. 2(c) departs from the true PSD by about 2 dB in the low-frequency region and more than 5 dB in the high-frequency region. Again, the reason for this is simply because noise PSD is highpass and significant to correlation-based power spectral estimators for this case (SNR = 5). This also indicates that correlation-based power spectral estimators are more sensitive to colored noise than to white noise. On the other hand, the AR spectral estimator associated with the proposed criteria is always unbiased no matter whether Gaussian noise is white or colored, but its variance is larger for colored noise than for white noise (see the dotted lines (average $\pm$ standard deviation) shown in Figs. 1(c) and 2(c)).

When the value of K is too small to include all nonzero terms $C_{M,x}(0,\ldots,0,k,k+i)$ in $c(i)$, the designed LPE filters and the associated AR spectral estimators are no longer unbiased, but their variance is also smaller for finite measurements. On the other hand, for the case that $x(n)$ is a narrow-band nonGaussian signal (corresponding to a large L (length of $h(n)$)), K must be chosen large enough for $\widehat{c}(i)$ to be unbiased, but the variance of $\widehat{c}(i)$ may become quite large in the meantime, which can result in very inaccurate LPE filter coefficients and the associated PSD estimates.

V. Conclusion

We have presented a new algorithm using two new cumulant-based MSE criteria $\tilde{J}_M$ and J_M given by (8) and (9), respectively, for the design of LPE filters. Delopoulos and Giannakis' third-order cumulant-based MSE criterion [3] is a special case of the proposed

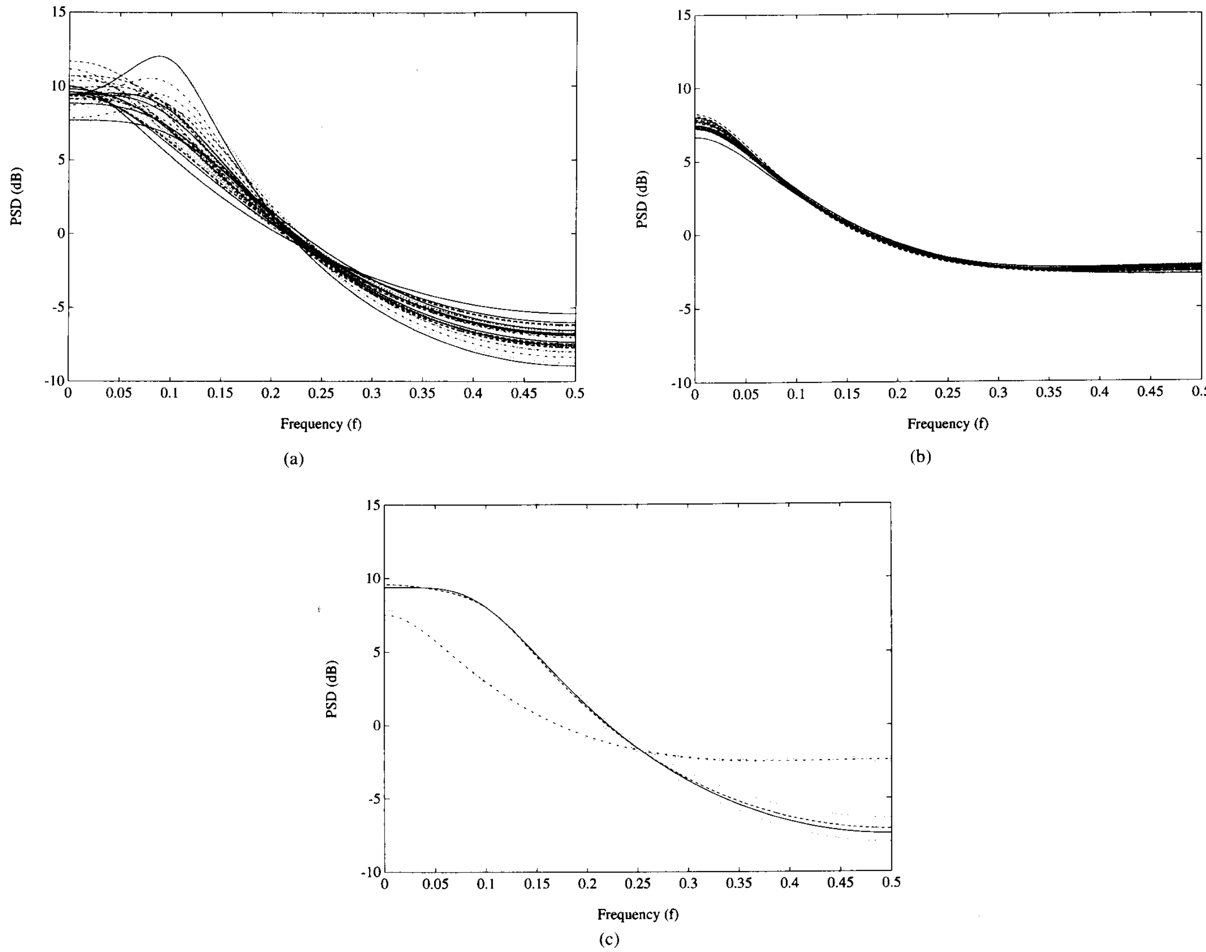

Fig. 2. Simulation results for the case of colored Gaussian noise with SNR = 5: (a) Thirty PSD estimates obtained by the proposed criterion $J_3(\tilde{J}_3)$; (b) those obtained by Burg's algorithm; (c) averages depicted by dashed line and dashed-dotted line as well as ± standard deviation (dotted line) of the 30 estimates shown in (a) and (b), respectively, together with the true PSD (solid line).

criterion for $M = 3$. The designed cumulant-based LPE filters with measurements corrupted by additive Gaussian noise are identical, and they are identical to the conventional (minimum-phase) LPE filter associated with the case that measurements are noise free (i.e., SNR $= \infty$) (see Theorem 1). When noise is nonGaussian, the designed LPE filter using Mth-order cumulants of measurements will no longer be noise insensitive if Mth-order cumulants of noise are significant compared with those of noise-free measurements. Moreover, coefficients of the designed cumulant-based LPE filters can be obtained by solving a set of symmetric Toeplitz linear equations to which the computationally efficient Levinson–Durbin recursion can be applied. Finally, some simulation results were presented to justify that the proposed cumulant-based MSE criteria can be used for the design of LPE filters.

We also found, from the performed simulation, that the power spectral estimator associated with the proposed criteria is unbiased and insensitive to both white and colored Gaussian noise, but its variance is larger than Burg's power spectral estimator, which has a bias crucially determined by noise power spectrum and SNR. However, for the case that $x(n)$ is a narrow-band nonGaussian linear process, when K is large, the variance of $\hat{c}(i)$ becomes quite large for finite measurements, which leads to large variance of the power spectral estimator associated with the proposed criteria. We leave the problem as a future research topic.

References

[1] S. M. Kay, *Modern Spectral Estimation.* Englewood Cliffs, NJ: Prentice-Hall, 1988.

[2] J. Makhoul, "Linear prediction: A tutorial review," *Proc. IEEE,* vol. 63, no. 4, pp. 561–580, Apr. 1975.

[3] A. N. Delopoulos and G. B. Giannakis, "Strongly consistent identification algorithms and noise insensitive MSE criteria," *IEEE Trans. Signal Processing,* vol. 40, no. 8, pp. 1955–1970, Aug. 1992.

[4] C.-Y. Chi, "Linear prediction, maximum flatness, maximum entropy and AR polyspectral estimation," *IEEE Trans. Signal Processing,* vol. 41, no. 6, pp. 2155–2164, June 1993.

[5] C.-Y. Chi and W.-T. Chen, "Linear prediction based on higher order statistics," in *Proc. Third Int. Symp. Signal Processing Applications ISSPA-92* (Gold Coast, Australia), Aug. 1992, pp. 230–233.

[6] ——, "Linear prediction based on higher order statistics by a new criterion," in *Proc. Sixth IEEE SP Workshop Stat. Array Processing* (Victoria, Canada), Oct. 1992, pp. 148–151.

[7] J. M. Mendel, "Tutorial on higher-order statistics (spectra) in signal processing and system theory: Theoretical results and some applications," *Proc. IEEE,* vol. 79, no. 3, pp. 278–305, Mar. 1991.

[8] C. L. Nikias and A. P. Petropulu, *Higher-Order Spectra Analysis.* Englewood Cliffs, NJ, Prentice-Hall, 1993.